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ABSTRACT

This study was undertaken to test the findings of Dienes and Jeeves concerning the strategies used by students to learn mathematical structures. The study also proposed to determine whether students are consistent across structures and embodiments in the strategies they use and the evaluations they give. One hundred adolescent girls were given three experimental tasks in game playing situations. In each task the student manipulated an apparatus that embodied a mathematical structure. The goal was to learn the rules of the game so as to make correct predictions about the outcome of each move. The first task, a Color Game, had been used by Dienes and Jeeves and was based on the Klein group. Task two, a Map Game, had a network structure. The third task, a Light Game, also embodied the Klein group Students' evaluations of how the game was played for the two group structure tasks fell into three categories--operator, pattern, or memory. Evaluations of the network structure game were categorized depending on whether the student focused on parts of the network or considered it as a whole. The sequence of a student's moves on each task was taken as a measure of the strategy she was using. Findings supported Dienes and Jeeves conclusions that the distribution of evaluations is ordered in decreasing frequency of occurrence as pattern, memory, operator and in decreasing efficiency as measured by the length of play as operator, pattern, memory. [Not available in hardcopy due to marginal legibility of original document.] (Author/CT)

A brief report of the study

# STRATEGIES IN LEARNING MATHEMATICAL STRUCTURES



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An important research study that included empirical observations of subjects learning mathematical structures and some evidence of regularities in their performance was carried out recently by Zoltan P. Dienes and Malcolm A. Jeeves (1965). In their study, strongly influenced by earlier work of Dienes (1959; 1960; 1963; 1964) and the studies of Bruner, Goodnow and Austin (1956) and Bartlett (1958), two groups of subjects -- adults and children -- were presented with the task of identifying the rules of games embodying mathematical group structures. Dienes and Jeeves classified the evaluations of the subjects into three groups and devised a coding system to identify strategies used. Evidence of the existence of a positive relationship between measured strategies and subjects' evaluations was reported. The study, Strategies in Learning Mathematical Structures was undertaken specifically to test their findings concerning strategies and to determine whether subjects are consistent across structures and embodiments in the strategies they use and the evaluations they give.

Each of the one hundred adolescent girls was given three experimental tasks in game-playing situations. The tasks were presented in three interviews with approximately two weeks between them. In each task the subject manipulated an apparatus that embodied a mathematical structure. The goal was to learn the rules of the game so as to make correct predictions about the outcome of each move. The interviewer kept track of the subject's moves in learning the game, the predictions she made, and the evaluations she gave of how the game worked.

The first task, a Color Game, had been used by Dienes and Jeeves and was based on the Klein group. The second task, a Map Game, had a network structure. The third task, a Light Game, also embodied the Klein group. A more complete description of the first and third tasks is given in Appendix A.

For the two group structure tasks, Dienes and Jeeves's scheme was used for classifying subjects' evaluations -- their views of how the game worked. Evaluations fell into three major categories, Operator, Pattern, or Memory, depending on whether subjects saw their moves as operators, looked for patterns in the plays, or merely attempted to memorize the plays. Evaluations of the network structure game were categorized as either Individual Roads or Detour Routes depending whether the subject focused on parts of the network or considered it as a whole. A more complete description of the Operator, Pattern and Memory Evaluations is given in Appendix B.

The sequence of a subject's moves on each task was taken as a measure of the strategy she was using, and strategy scores were calculated from this sequence. Dienes and Jeeves's system was used for the two group structure tasks. A strategy scoring system was devised for the network structure task. A more complete description of Dienes and Jeeves's strategy scoring system is given in Appendix C.

Distributions and cross-tabulations of the evaluations and the strategy scores confirmed Dienes and Jeeves's findings that the distribution of evaluations is ordered in decreasing frequency of occurrence as Pattern, Memory, Operator, and in decreasing efficiency, as measured by the mean length of play, as Operator, Pattern, Memory. Other findings by Dienes and Jeeves, such as zero correlation between measures of performance and intelligence and the existence of a relationship between evaluations and strategies as measured by their scoring system, were not supported. Consistency across tasks was most pronounced for measures of success and failure. Evaluations showed some consistency across tasks, especially across the two group structure tasks. Strategies tended not to be consistent across tasks, probably owing to inadequacies in the strategy scoring system. Appendix D contains more information on some of these findings.

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## Appendix A

### The Klein Group Tasks

Mathematically, each of the tasks can be considered an operational system. That is, each task consists of a set of elements on which a closed binary operation is defined. During each task, each play a subject performs represents one outcome of the binary operation in the system. The subject's goal is to learn all the rules of the game: that is, the subject is to learn the outcome of every possible combination of two elements in the set.

#### Color Game

The Color Game consists of two identical sets of four cards, and a board with a window in it that can be opened or closed from behind. The window is constructed so that a card can be displayed in it. Each set of cards contains a yellow card, an orange card, a blue card, and a green card. The interviewer and the subject each have a set of cards. One of the interviewer's four cards is initially displayed in the window. The subject plays one of his cards by placing it in front of the window, and the interviewer then closes the window and opens it again with one of his four cards showing. Mathematically, the Color Game possesses the structure of the Klein group, the four-element group in which each element is its own inverse. The elements of the group are the colors of the four cards and the binary operation is playing a color against a color appearing in the window. The outcome of the binary operation is the color of the card that next appears in the window, and this depends on the combination of the color of the card played and the color of the card in the window previously. Yellow is the identity element. The outcomes of all the binary combinations are illustrated in Table 1.

TABLE 1

OUTCOMES OF PLAYING A CARD AGAINST
A CARD IN THE WINDOW

| Card Played | Card in the Window | | | |
|---|---|---|---|---|
| | Yellow | Orange | Blue | Green |
| Yellow | Yellow | Orange | Blue | Green |
| Orange | Orange | Yellow | Green | Blue |
| Blue | Blue | Green | Yellow | Orange |
| Green | Green | Blue | Orange | Yellow |

Light Game

The Light Game consists of a closed wooden box, 16 inches square by 2 inches deep. A light bulb and a four-pole double throw switch are located in each corner of the upper face of the box. The bulb-switch combinations are labeled Saturn, Mars, Venus, and Jupiter. The electrical wiring for the game is enclosed in the box, with only a single wire and plug exposed.

One of the four light bulbs is lit initially. The subject switches any one of the four switches by moving the switch from one pole to the other. This causes the light bulb that is lit to go out, and then one of the four light bulbs to light.

Mathematically the Light Game is isomorphic to the Color Game. The elements are the sets of lights and switches and the binary operation is the combination of the set that is switched with the set that is lit. The outcome of the binary operation is the set that lights next, and this depends on the particular combination of the set that was switched and the set that was lit previously. The set Saturn plays the identity role, which Yellow plays in the Color Game, and Mars, Venus, and Jupiter behave in the same way as Orange, Blue, and Green, respectively. The outcomes of all the possible combinations are illustrated in Table 2.

TABLE 2

OUTCOMES OF SWITCHING A SET
WHEN A SET IS LIT

| Set Switched | Set Lit | | | |
|---|---|---|---|---|
| | Saturn | Mars | Venus | Jupiter |
| Saturn | Saturn | Mars | Venus | Jupiter |
| Mars | Mars | Saturn | Jupiter | Venus |
| Venus | Venus | Jupiter | Saturn | Mars |
| Jupiter | Jupiter | Venus | Mars | Saturn |

## Appendix B

## Evaluations

The evaluations of the group structure games were classified into the Operator, Pattern, or Memory types, or combinations of these, according to the guidelines established by Dienes and Jeeves. Dienes and Jeeves discovered that for the Four-element group structures, their subjects fell into one or a combination of the three evaluation types defined as follows:

1. Operational Type

   These subjects appeared to regard the card played as operating on the card in the window, having the power to alter this card.

2. Pattern Type

   These subjects appeared to regard the game as divided up into a certain number of sub-sections. They regarded the card in the window and the card on the table as on the same level. They described one part of the table at a time as hinging together into a pattern. The whole table appeared to them, by their reports, as a large pattern put together out of certain smaller patterns.

3. Memory Type

   These subjects stated that they merely memorized all the different combinations.[1]

Examples of statements included in a pure Operator evaluation for the Color Game are:

The Yellow Card doesn't change the color of the card in the window. The Orange Card changes Orange to Yellow, Yellow to Orange, Blue to Green, and Green to Blue.

Similar statements for the Light Game are:

The Saturn switch doesn't change the set that is on. The Mars switch changes Saturn to Mars, Mars to Saturn, Venus to Jupiter and Jupiter to Venus.

Because of the arrangement of the lights and switches in the corners of the square, examples of alternative Operator statements for the Light Game are:

Saturn keeps the same light on.
Mars changes the light to the adjacent horizontal one.
Jupiter changes the light to the diagonal one.

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1. Zoltan P. Dienes and Malcolm A. Jeeves, Thinking in Structures, (London: Hutchinson Educational, 1965), p. 36.

A pure Pattern evaluation, for both the Color Game and the Light Game, consists of breaking the game into three patterns that describe the following regions of the table defining the binary operation: the S region, the same element both played and displayed; the C region, the identity element combined with one of the other elements either played or displayed; and the T region, the combination of two distinct elements neither of which is the identity element. Both four-groups possess the property of commutativity. That is, the result of the combination of two elements is the same regardless of which is played and which is displayed. Thus the Pattern evaluations do not differentiate the card or set played from that displayed. In the Klein group, any combination in the S region produces the identity element, any combination in the C region produces the element of the pair that is not the identity element, and any combination of the two distinct non-identity elements in the T region produces the third element that is not the identity.

Examples of statements included in a pure Pattern evaluation for the Color Game are:

The Yellow card in combination with any other colored card results in the other color appearing in the window. (This represents the C-section of the table.) The combination of two of the same colored cards results in yellow appearing in the window. (This is the S-section.)

For the Light Game, examples are:

Two of the same planets result in Saturn.
Any two of Mars, Venus, and Jupiter results in the third one. (This is the T-section.)

A pure Memory evaluation is simply the stating of all sixteen binary combinations and their outcomes. If the commutativity property is discovered, the number of combinations reduces to ten. Very few of the subjects who did not give Operator or Pattern evaluations were able to memorize all the combinations correctly. Many subjects gave evaluations which were combinations of Memory evaluations and Pattern or Operator evaluations. That is, they saw one or two patterns or operators (in most cases they saw the role of the identity element) and memorized all the other combinations. Subjects who gave a combination evaluation were classified as such, for example, Pattern-Memory or Operator-Pattern.

Every subject was classified according to what she said, whether she was a success or a failure. Some of the subjects who failed memorized only a few of the combinations and their evaluations were classified as pure Memory. Others, notably those who became frustrated and asked to stop, gave stories or remarks that did not fit any definable category. These evaluations were classified together as Other. Usually these remarks contained references to cycles or sequences of outcomes or asserted that the interviewer had changed the rules in midstream. The distribution of evaluations for the Color Game and the Light Game are given in Table 3.

TABLE 3

DISTRIBUTION OF EVALUATIONS FOR THE
COLOR GAME AND THE LIGHT GAME

| Evaluations | Color Game | Light Game |
|---|---|---|
| Operator | 0 | 2 |
| Operator-Pattern | 13 | 13 |
| Operator-Pattern-Memory | 0 | 3 |
| Operator-Memory | 1 | 4 |
| Pattern | 23 | 20 |
| Pattern-Memory | 15 | 20 |
| Memory | 37 | 28 |
| Other | 11 | 10 |
| Total | 100 | 100 |

Appendix C

## Strategy Scores

Dienes and Jeeves developed a scoring system to measure the behavior of subjects playing a game based on a group structure. They devised the following definitions for an Operator score and a Pattern score to account for the corresponding evaluations given by the subjects:

> Operator Score. This is obtained by counting the total number of cards played in runs of three or more of the same card being played in succession, this number being divided by the total number of freely selected instances.[1]

> Pattern Score. Runs of three or more combinations from the same section are counted towards the pattern score. Runs of three or more combinations from the same section are counted if interspersed by single correct predictions from other sections. It is not possible to generate any desired instance immediately after any given instance... For these reasons a sectional run is still counted as a run towards the pattern score if it is interspersed by single instances correctly predicted, which do not belong to the section from which the run is taken. This applies naturally to every section, i.e. to C, S, and T.[2]

In explanation of these definitions, Dienes and Jeeves say that:

> It must be remembered that self-evaluations...are not necessarily actual stretegies used by the subjects. In order to check how far these evaluations coincide with strategies used by the subjects, the operator and pattern scores... were used. An optimum operator strategy would be to play the same card a number of times until it was discovered what kind of operation this particular card induced on the window. This is why long runs from three upwards were scored as part of the operator score. Similarly the pattern scoring was done on how far the subjects kept to the same part of the table while solving their task.[3]

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1. Dienes and Jeeves, p. 25.
2. Ibid., p. 27.
3. Ibid., p. 37.

Appendix D

The Relationship of Evaluations and Strategy Scores

Underlying the present experiment was the expectation that there would be a relationship between the evaluations and the strategy scores for successful subjects in the Color Game. It was also expected that this relationship would be found in the Light Game since its structure is isomorphic to that of the Color Game and it is played in a similar manner.

Dienes and Jeeves found "considerable evidence of the existence of a positive relationship between measured strategies and subjects' evaluations."[1] However, they do not report this evidence fully. They note that Operator evaluations and Operator scores were related at the

> 4% level as measured by one-tail t-test....In the operational group [subjects who gave either an entirely operational evaluation or a partly operational one] seven out of eight subjects scored above the median of the operator scores for all subjects. In the non-operational group seven out of twenty-one scored above the median.[2]

They reported similar findings for the relationship between Pattern or partial Pattern evaluations and Pattern scores, but no data at all were given. To determine the relationship between evaluations and scores in both the Color Game and the Light Game, the methods used by Dienes and Jeeves were followed. Successful subjects who gave either a pure or partial Operator evaluation were differentiated from those successful subjects who did not, and both groups were divided at the median of the Operator scores. The same procedure was used in the case of the Pattern or partial Pattern evaluations and the Pattern scores. In all cases there were no significant differences found.

Consistency of Evaluations

In order to see if subjects were consistent in their evaluations across the group structure tasks, a distribution of all subjects' evaluations was compiled (see Table 4).

The table was considered to consist of three parts. The cells that make up the principal diagonal depict the total number of subjects who were consistent in their evaluations across the group structure tasks; the cells below this diagonal depict the total number of subjects who gave a higher-order evaluation (the evaluations were ordered Operator, Pattern, Memory in order of decreasing efficiency); and the remaining cells, those above the diagonal, depict the total number of subjects who gave a lower-order evaluation. The sums of the numbers in these parts, along with the sums of the expected frequencies determined from Table 4, are shown in Table 5. The results illustrate that there is considerable consistency in subjects' evaluations.

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[1]Dienes and Jeeves, p. 75.

TABLE 4

EVALUATIONS OF ALL SUBJECTS ACROSS
THE COLOR GAME AND THE LIGHT GAME

| Color Game Evaluations | Light Game Evaluations | | | | Total |
|---|---|---|---|---|---|
| | Operator or Partial Operator | Pure Pattern | Memory or Pattern-Memory | Other | |
| Operator or Partial Operator | 8 | 2 | 4 | 0 | 14 |
| Pure Pattern | 4 | 12 | 7 | 0 | 23 |
| Memory or Pattern-Memory | 9 | 5 | 31 | 7 | 52 |
| Other | 1 | 1 | 6 | 3 | 11 |
| Total | 22 | 20 | 48 | 10 | 100 |

TABLE 5

CALCULATION OF CHI-SQUARE FOR CONSISTENCY OF EVALUATIONS
ACROSS THE COLOR GAME AND THE LIGHT GAME

| Evaluations | Frequency | |
|---|---|---|
| | Observed | Expected |
| Consistent on Light Game | 54 | 33.8 |
| Higher Order on Light Game | 26 | 36.8 |
| Lower Order on Light Game | 20 | 29.4 |
| Total | 100 | 100.0 |

$\chi^2 = 18.2, p < .001$

## Consistency of Strategy Scores

Although the strategy scores were not related to the evaluation, consistency of the strategy scores across the group structure tasks was still analyzed.

A product-moment correlation of the data gave correlations of 0.45 for Operator scores and 0.14 for Pattern scores for all subjects across the Color Game and the Light Game. When the scores of the successful subjects were analyzed, the correlations were 0.14 and -0.18 respectively. These results indicate that the Operator scores of all subjects across the Color Game and the Light Game tend to be moderately correlated.

Further analysis showed that of the 32 subjects scoring above the median of the Operator score for each group structure task, only 7 were successful on both. Thus subjects who used an Operator strategy on both tasks were more likely to have failed on one or both than to have succeeded consistently. Although the Operator evaluations were found to be the most efficient, high Operator scores do not indicate efficiency in terms of performance. On the contrary, the relationship is in the opposite direction.

DOCUMENT RESUME

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ABSTRACT

The research reported dealt with the ability of first, second, and third grade students to understand the topological concepts of solids, sheets, lines, networks, and of order and betweenness. The concepts were presented through the use of manipulative materials. The instruction was conducted by the regular teachers for a period of three weeks after which test items in workbooks completed by the students, were used for obtaining data. The achievement of each class on each item was tabulated, and differences in performance are discussed. The author concludes that the material presented seems suitable for children in the age range studied. (JG)

NC

Early





# Selected Topological Concepts Taught to Children, Ages Six to Nine

Sair Ali Shah, University of Georgia, 1971

## Introduction

The reactions of some children in grades one through three were obtained, to some concepts in "Intuitive Topology," when three teachers in Georgia taught them a series of lessons, over a period of three weeks. An assessment of performances was made from workbooks provided for the children and it was found that a reasonable standard of attainment was achieved.

## Mathematical Content Taught

1. Some invariant properties of an object under the operations of bending and stretching, without cutting or joining. This is similar to a body undergoing "elastic motion" and observing the properties, which remained the same. These properties may be considered as some topological properties of an object. Here we considered the topological equivalences of (a) solids, (b) sheets, (c) "lines" and "networks".
2. We also included: Order and Betweenness, which was illustrated by "order of objects on a string."

## Psychological Bases for the Study

1. We used the idea of "proceeding from the concrete to the abstract." Children were provided with kits, which contained: four ounces of plastic clay, ten colored beads and six pieces of toy-pipe cleaners. The material was used by the children for making models and comparing them for their invariant features (that is, features which remained the same) after bending or stretching without cutting or joining.
2. Piaget's idea of Reversibility, was used as a basis for the activities provided. Children were encouraged to carry out the operation of 'bending and stretching without cutting or joining'. These exercises provided 'movement of thinking' at the concrete level. Opportunities at the abstract level were provided in the workbook.
3. Piaget's idea of conservation was used as the basis for obtaining the invariant features of an object under the operation of 'bending and stretching without cutting or joining. The child had to consider the features, which were conserved under the operation.

Procedure

1. Samples: (a) three teachers at the same school taught the lessons. One taught the grade one, another the grade two, and the next the grade three class. All three teachers were receptive to the ideas of the content and cooperated greatly in using the material with the children. (b) the sample of eighty-eight(88) pupils were within the age range six to nine years. They were distributed as follows: 29 in grade 1 (6+), 29 in grade 2 (7+) and 30 in grade 3 (8+).
2. Teaching the Content
   A teacher's guide was prepared for assisting with instruction. The teachers were given a forty-five minute session by the author, on the content and technique of teaching to be used. The general outline of the technique of teaching had the following stages:
   I. Free play with material from children's kit. Let them make models.
   II. Comparison of models for common features.
   III. Look at pictures in workbook and compare them for common features.
   IV. Ask questions to direct attention to common features of models. The workbook provided numerous exercises which the children worked on.
3. Instrument for Obtaining Estimates of Achievement
   Some of the exercises in the workbook were tests. We analyzed the responses in the workbooks, but scores were obtained only for the test exercises.
   The material in the workbooks can be broadly classified into two areas: (a) exercdses to find out what criteria children of this experiment used for comparing the given pictures, (b) exercises to find out if they can 'match' objects with common features (or elements) after bending or stretching without cutting or joining.
   The invariant features included in the pictures given in the exercises were: (a) objects without holes, (b) objects with the same number of holes, (c) networks with the same distribution of beads and strings, (d) beads in the same order on two or more strings, (e) closed paths, (f) open paths.

Activities and Results

I. During "free play" with the material of the kit some of the models made by the children were.

1. For solids, they made (a) animals - dogs, cats, rabbits - all with four legs, (b) doughnuts, car tires, rings(they were asked to make things with one hole).
2. For 'sheets' they were told to make a flat sheet with the plastic clay, and then make things with it by bending and stretching it. They made cookies, plates, bowls, and hats.
3. For 'lines' they were asked to bend their toy pipe cleaners and make letters. They made the letters. C, S, N, Z.
4. For "networks" they used beads for joining pieces of 'pipe cleaners'. Beads were also placed at the free ends of the pipe cleaners.
5. For order of things on a string, they placed four or five beads, each of a different color on a 'pipe cleaner.' The pipe cleaner was then bent, and the order of the beads were compared by observing which bead followed each other.

The above activities, which were performed with concrete material were satisfactorily done, and children discovered without difficulty, the common features of two models. For example. a dog and a cat each has four legs; a plate and a bowl are both made from a sheet; the letters C and S are made by bending one piece of wire; two similar networks have the same number of beads and strings; the order of the beads on a string, was the same, in spite of bending the string.

II. The next activity was that the children were given pictures to be compared. Here it was expected that they will use the invariant features for explaining how the pictures are the same. Estimates of their performances were obtained from their workbooks. Children did find out what these features were and used them. The features were (a) objects without holes, (b) objects with the same number of holes, (c) networks with the same distribution of beads and strings, (d) beads in the same order in two or more strings, (e) closed paths, (f) open paths.

Other criteria children used for comparison were. shape, size, smoothness, joined, not joined, twisted.

III. Responses to test items (see appendix) were obtained here. The areas covered were:

(a) Comparing objects for common features, after bending and stretching, without cutting or joining. These included: solids, sheets, and wires. Items 1 to 6 inclusive of the test covered this area.

(b) Matching networks with the same number of beads and segments. This was item 7 of the test.

(c) Testing Order and Betweenness, which was covered in item 8 of the test.

Percentages of correct responses were obtained for each of the different test areas as shown below.

Sample: Gd. I = 29, Gd. II = 29, Gd. III = 30

| ITEMS ON TEST | PERCENT GAINED BY Gd. I | II | III |
|---|---|---|---|
| I. Bending and Stretching without cutting or joining of: | | | |
| (a) Solids | | | |
| Item 1 (See test in Appendix) | 58 | 71 | 97 |
| Item 2 | 61 | 69 | 71 |
| Item 3 | 81 | 33 | 77 |
| Item 4 | 52 | 27 | 71 |
| (b) Sheets | | | |
| Item 5 | 81 | 71 | 87 |
| (c) Wire | | | |
| Item 6 | 68 | 71 | 80 |
| II. Networks Item 7 | 37 | 68 | 97 |
| III. Order and Betweenness. Item 8 | 52 | 74 | 79 |

Items 1,2,3,4 covered exercises on the topological equivalent of solids, using number of holes as the criterion. On the whole more than fifty percent of the children, except in certain cases of the grade 2 class, gained scores of 50 percent and more. If we consider 50 percent as our level of satisfactory performance, then the responses were reasonable.

Item 5, which involved the bending and stretching of a sheet was used to estimate the topological equivalence of sheets. The percentage ranged from 71 for grade 2, to 87 for grade 3. Grade 1 percent was 81. These were well over our criterion level.

Item 6 was used for the topological equivalence of wires or segments. The percentages ranged from 68 to 80, which were above the criterion level.

Item 7 on the topological equivalence of networks had percentages: 37 for grade 1, 68 for grade 2, and 97 for grade 3. Grades 2 and 3 were above the criterion level, but grade 1 was not.

Item 8 on Order and Betweenness had percentages. 52 for grade 1, 74 for grade 2, and 79 for grade 3. These were above the criterion level.

## Discussion

Among the several variables affecting the scores, were teacher competence, time spent by children on the material, and understanding of material, which involves reading competence of children and understanding of mathematical concepts.

With respect to teacher competence, a teacher's guide was provided and a forty-five minute orientation session was given. Though the teachers expressed understanding of the material, it was felt that since the topic was new, they would have felt more confident, if they had a course on "intuitive topology."

Concerning the time spent on the program by the children, Grade III had ten half-hour sessions. Their performance was satisfactory. The Grade II class spent fifteen twenty minute sessions. Their scores were the lowest in many cases of the three grades and this may be because of the shorter sessions. Another factor, which contributed to the low scores of Grade II, was the high rate of absentism. Grade I spent fifteen half-hour sessions and in some cases gave better performances than the Grade II class.

For understanding of the material, some of the reading involved was beyond the Grade I level. Thus this affected the scores. The teacher of the Grade I class explained that the oral responses of her class were better than their written answers. The scores of the Grade II class were also affected by the reading involved. They scored better on the non-verbal items.

On the scores themselves, using the criterion level of obtaining a score of at least fifty percent on each exercise, the distribution of all the grades except in a very few cases, were satisfactory.

From the above we conjecture that the material, which was prepared by using some psychological bases, seems suitable for children in the age range six to nine years.

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2. B.H. Arnold, Intuitive Concepts in Elementary Topology, Prentice-Hall, 1963.

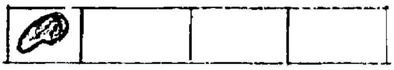
3. Some Lessons in Mathematics, Cambridge University Press.

4. Wolfgang Franz, General Topology, Fred. Ungar. Publishing Co. N.Y.

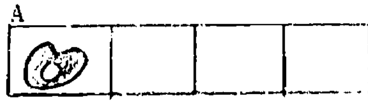
## APPENDIX I TEST ITEMS

1. Draw different things which we can make from picture A by bending and stretching without cutting or joining.

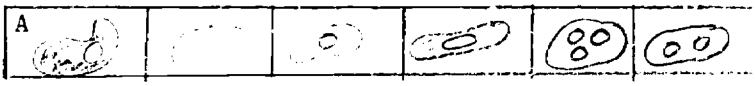
a) A



b) A

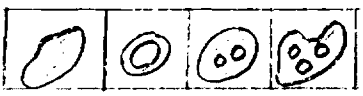


2. Draw a mark around the things which we can make from picture A by bending or stretching without cutting or joining.

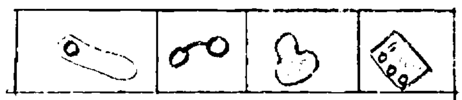



3. Match the one in A to the one in B which you can get by bending or stretching without cutting or joining.

A

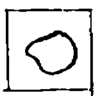


B



4. Can you get picture B from picture A by bending or stretching without cutting or joining? Put a ring around: yes, no, or I don't know.

A



B

yes
no
I don't know

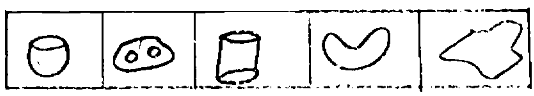
A

B

yes
no
I don't know

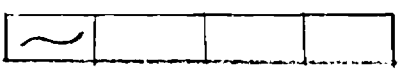
5. Draw a line from picture A to all the pictures which you can make by bending or stretching but not cutting or joining.

A

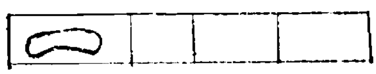


6. Draw three things which you can make by bending or stretching picture A, but not cutting or joining.

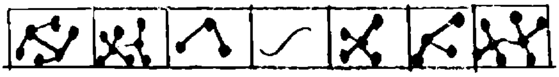
A

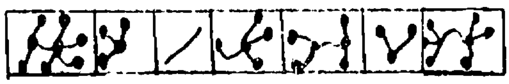


A



7. Match the pictures which are the same from the two sets.





8. Draw the missing things to make all of these pictures look the same as picture A.

A